

QO'ZG'ALMAS QISMI TRILAPLASIANDAN IBORAT BO'LGAN DISKRET SHREDINGER OPERATORGA MOS FREDHOLM DETERMINANTINING XOSSALARI

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Annotatsiya. Mazkur ishda bir o'lchovli \mathbb{Z} panjarada aniqlangan $\lambda, \mu \in \mathbb{R}$ parametrga ega $\hat{h}_{\lambda\mu}$ diskret Shredinger operatorga mos Fredholm determinantining xossalari tadqiq etilgan. Bu xossalarni o'rganish $\hat{h}_{\lambda\mu}$ operatorning spektrlarini o'rganishda qulay hisoblanadi. Operatorning qo'zg'almas qismi diskret trilaplasiyan operatori bilan, lokal bo'lmagan potensial esa kroneker delta funksiyasi va siljitish operatori bilan tavsiflanadi. Muhim spektrdan chapda xos qiymatlarning mavjudlik bo'lishining zaruriy sharti va uning parametrlarga bog'liqligi isbotlanadi. Shuningdek Fredholm determinantining koeffitsiyentlari uchun turli asimptotikalar olinadi.

Kalit so'zlar. Diskret Shredinger operatori, trilaplasiyan, koordinata tasvir, impuls tasvir, delta potensial, muhim spektr, xos qiymat, trivial yechim.

KIRISH

Panjaradagi aniqlangan Shredinger operatorlari 1990-yillarda D. Mattis, A. Mogilner va J. Rauchlar tomonidan o'rganila boshlagan bo'lib, bu yo'nalishdagi ilmiy izlanishlar tez sur'atlar bilan rivoj topdi. Diskret Shredinger operatorlarining spektral xossalari xalqaro miqyosda E. Korotyaev, N. Saburova, G. Berkolaiko, R. Carlson, S. Fulling, P. Kuchment, F. Chung, H. Cycon, R. Froese, W.Kirsch, B.Simon, Sh. Birman, M. Solomjak, F. Hiroshima, I. Sasaki, T. Shirai, A. Suzuki kabi olimlar tomonidan o'rganilgan.

Ushbu maqolada \mathbb{Z} panjaradagi $x_0, -x_0$ nuqtalarida berilgan nolokal potensialga ega Shredinger operatorining spektrini o'rganishda muhim o'rin tutadigan Fredholm determinantining xossalarini tahlil qilishni maqsad qilganmiz. Shredinger operatorlarining spektral xossalarini o'rganish matematik fizika va operatorlar nazariyasida muhim tadqiqot yo'nalishi bo'lib kelmoqda. Shuningdek, Operatorlarning spektral nazariyasini o'rganish operatorlar bilan bog'liq fizik jarayonlarni yaxshiroq tushunishga imkon beradi. Muhim spektrdan tashqarida operatorning xos qiymatlari mavjudligini va ularning $\mu \in \mathbb{R}$, $\lambda \in \mathbb{R}$, $x_0, -x_0 \in \mathbb{Z}$ o'zaro ta'sir parametrlariga bog'liqligini 3.1-teoremada aniq ko'rsatamiz.

ADABIYOTLAR SHARHI

Panjaradagi Shredinger operatorlarining spektral xossalarini va xos qiymat xarakteristikalarini o'rganish ko'plab ishlarda muhokama qilingan. Jumladan, respublikamiz olimlaridan S. Laqayev, A. Xalxo'jayev, J. Abdullayev, Z. Muminov, Sh. Alladustov, Sh. Laqayev, Sh. Xolmatov, M. Pardaboyev, A. Boltayev, H. o'zlarining ilmiy ishlarida ko'rishimiz mumkin. Xususan [1]-[2] ishlarda d o'lchamli va 3 o'lchamli panjarada berilgan, qo'zg'almas qismi standart laplasiandan va potensial qismi $x_0 \in \mathbb{Z}^d$ ($x_0 \in \mathbb{Z}^3$) nuqtada ta'sir etuvchi Kroneker delta funksiyasi bilan tavsiflanadigan diskret Shredinger operatorlarning spektral va bo'sag'a tahlillari o'rganilgan. [3]-[4],[8] ishlarda esa bir o'lchamli panjarada berilgan laplasiandan va Kroneker delta funksiyasi bilan tavsiflanadigan diskret Shredinger operatorlarning xos qiymat asimptotikalari o'rganilgan.

METODOLOGIYA

Aytaylik \mathbb{Z} - bir o'lchamli panjara, $\mathbb{T} = (-\pi, \pi]$ - esa bir o'lchamli tor, $\ell_2(\mathbb{Z})$ - \mathbb{Z} panjarada aniqlangan kvadrati bilan jamlanuvchi funksiyalarning Hilbert fazosi va $L_2(\mathbb{T})$ esa \mathbb{T} torda aniqlangan kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosi bo'lsin.

Mazkur ishda biz, $\ell_2(\mathbb{Z})$ Hilbert fazosida berilgan trilaplasiyan operatori va delta-potensialdan tashkil topgan

$$\hat{h}_{\lambda\mu} = -\Delta^3 - \hat{V} \quad (1)$$

ko'rinishdagi Shredinger tipli operatorning spektral xossalarini tahlil qilamiz. Bunda, Δ^3 - standart diskret Laplasianning uchinchi iteratsiyasi ya'ni trilaplasiyan \hat{V} esa $x_0, -x_0 \in \mathbb{Z}$ nuqtalarda berilgan nolokal potensialdir:

$$\hat{V} = \lambda V_0 + \frac{\mu}{2}(V_0 T(x_0) + T^*(x_0)V_0) + \frac{\mu}{2}(V_0 T(-x_0) + T^*(-x_0)V_0) \quad (2)$$

bunda V_0 esa $\ell^2(\mathbb{Z})$ dagi Kroneker delta funksiyasi $\delta[\cdot, 0]$ ga ko'paytirish operatoridir:

$$(V_0)f(x) = \delta[x, 0]f(x)$$

Odatda (1) ko'rinish operatorning koordinata tasviri deyiladi. Bu operatorning spektral xossalarini o'rganish uchun qulaylik maqsadida Fur'e almashtirishlaridan foydalanib, uning koordinat tasviridan impuls tasviriga o'tib olamiz.

Aytaylik \mathcal{F} - standart Fur'e almashtirishi bo'lsin: $\mathcal{F}: L_2(\mathbb{T}) \rightarrow \ell_2(\mathbb{Z})$, uning $f \in L_2(\mathbb{T})$ elementga ta'siri quyidagicha:

$$(\mathcal{F}f)(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{T}} f(p)e^{-ixp} dp, \quad x \in \mathbb{Z};$$

\mathcal{F}^{-1} esa teskari Fur'e almashtirishi bo'lsin: $\mathcal{F}^{-1}: \ell_2(\mathbb{Z}) \rightarrow L_2(\mathbb{T})$

uning $\hat{f} \in \ell_2(\mathbb{Z})$ elementga ta'siri quyidagicha:

$$(\mathcal{F}^{-1}\hat{f})(p) = \frac{1}{\sqrt{2\pi}} \sum_{x \in \mathbb{Z}} \hat{f}(x)e^{ixp}, \quad p \in \mathbb{T}.$$

$h_{\lambda\mu}$ diskret Shredinger operatorining impuls tasviri quyidagicha ifodalanadi:

$$h_{\lambda\mu} = h_0 - V_{\lambda\mu} \quad (3)$$

bunda $h_0 := \mathcal{F}^{-1}(-\Delta^3)\mathcal{F}$ operator $L_2(\mathbb{T})$ Hilbert fazosida haqiqiy qiymatli $\varepsilon(p) = (1 - \cos p)^3$ funksiyasiga ko'paytirish operatori hisoblanadi:

$$(h_0 f)(p) = \varepsilon(p)f(p) = (1 - \cos p)^3 f(p), \quad f \in L_2(\mathbb{T})$$

$e(\cdot)$ funksiyasi \mathbb{T} da haqiqiy qiymatli funksiya bo'lib, fizik adabiyotlarda Laplas operatorining dispersiya munosabati deb ataladi. $V_{\lambda\mu} := \mathcal{F}^{-1}(\hat{V})\mathcal{F}$ qo'zg'alish esa $f \in L_2(\mathbb{T})$ ga rangi ikkiga teng bo'lgan integral operator ko'rinishida ta'sir etadi:

$$\begin{aligned} (V_{\lambda\mu} f)(p) &= \\ &= \frac{1}{2\pi} \int_{\mathbb{T}} \left[\lambda + \frac{\mu}{2} (e^{i(x_0,p)} + e^{i(-x_0,p)} + e^{i(x_0,q)} + e^{i(-x_0,q)}) \right] f(q) dq = \\ &= \frac{1}{2\pi} \int_{\mathbb{T}} [\lambda + \mu(\cos(x_0,p) + \cos(x_0,q))] f(q) dq, \quad f \in L_2(\mathbb{T}). \quad (4) \end{aligned}$$

TAHLIL VA NATIJALAR

$h_{\lambda\mu}$ operatorning xossalari. Operatorlar nazariyasiga ko'ra Hilbert fazolarida aniqlangan $h_{\lambda\mu}$ operatorning spektrlarini o'rganishda bu operatorning xossalarini (chiziqililigini, chegaralanganligini, o'z-o'ziga qo'shmaligini) bilish juda muhimdir.

1-Lemma. $h_{\lambda\mu}$ o'z-o'ziga qo'shma operator.

Isboti: $h_{\lambda\mu}$ operatorning o'z-o'ziga qo'shmaligi $\varepsilon(p)$ funksiyaning haqiqiy qiymatli funksiya ekanligidan, $V_{\lambda\mu}$ operatorning chekli o'lchamli ekanligidan hamda o'z-o'ziga qo'shmalik ta'riflaridan bevosita kelib chiqadi.

h_0 ning qo'zg'alishi V_{x_0} ikki o'lchovli operator ekanligidan muhim spektrning turg'unligi haqidagi Veyl teoremasiga [6]-[7] ko'ra quyidagi lemma o'rinli:

2-Lemma. h_{μ} operatorning muhim spektri quyidagi kesma bilan ustma-ust tushadi:

$$\sigma_{ess}(h_{\mu}) = \sigma(h_0) = \sigma_{ess}(h_0) = [\varepsilon_{min}, \varepsilon_{max}] = [0,8].$$

Fredholm determinant. Avvalo $z \in \mathbb{C} \setminus [0,8]$ qiymatlari uchun quyidagi belgilashlarni kiritib olamiz:

$$\begin{aligned} \alpha(z) &= \frac{1}{2\pi} \int_{\mathbb{T}} \frac{1}{\varepsilon(t) - z} dt, \quad \beta(z) = \frac{1}{2\pi} \int_{\mathbb{T}} \frac{\cos(x_0, t)}{\varepsilon(t) - z} dt, \\ \gamma(z) &= \frac{1}{2\pi} \int_{\mathbb{T}} \frac{\cos^2(x_0, t)}{\varepsilon(t) - z} dt, \quad \xi(z) = \beta^2(z) - \alpha(z)\gamma(z). \end{aligned}$$

$z \in \mathbb{C} \setminus [0,8]$ da $h_{\lambda\mu}$ ($\lambda, \mu \in \mathbb{R}$) operatorga mos Fredholm determinanti

$$D(\lambda, \mu, z) = \mu^2 \xi(z) - 2\beta(z)\mu - \lambda\alpha(z) + 1 \quad (5)$$

ko'rinishga ega bo'ladi.

1-teorema. $z \in \mathbb{C} \setminus [0,8]$ soni $h_{\lambda\mu}$ operatorning xos qiymati bo'lishi uchun, $D(\lambda, \mu, z) = 0$ bo'lishi zarur va yetarli.

Isboti: (Zaruriyligi) Aytaylik, $z \in \mathbb{C} \setminus [0,8]$ soni $h_{\lambda\mu}$ operatorning xos qiymati bo'lsin, U holda $z \in \mathbb{C} \setminus [0,8]$ soni $(h_{\lambda\mu} f)(p) = z f(p)$ xos qiymat tenglamasini qanoatlantiradi. Ya'ni

$$[\varepsilon(p) - z]f(p) = \frac{\lambda}{2\pi} \int_{\mathbb{T}} f(s) ds - \frac{\mu \cos(x_0, p)}{2\pi} \int_{\mathbb{T}} f(s) ds +$$

$$+ \frac{\mu}{2\pi} \int_{\mathbb{T}} \cos(x_0, s) f(s) ds$$

munosabatga ega bo'lamiz. Bunda

$$C_1 = \frac{1}{2\pi} \int_{\mathbb{T}} f(s) ds, \quad C_2 = \frac{1}{2\pi} \int_{\mathbb{T}} \cos(x_0, s) f(s) ds$$

belgilashlar kiritib C_1 va C_2 larga nisbatan chiziqli tenglamalar sistemasiga kelamiz.

$$\begin{cases} (1 - \mu b(z) - \lambda a(z))C_1 - \mu a(z)C_2 = 0 \\ (-\lambda b(z) - \mu c(z))C_1 + (1 - \mu b(z))C_2 = 0. \end{cases} \quad (6)$$

(6) sistema bir jinsli sistema bo'lganligi sababli, nolmas yechimga ega bo'lishi uchun asosiy determinanti nolga teng bo'lishi zarur edi. Asosiy determinantni $D(\lambda, \mu, z)$ deb belgilasak,

$$D(\lambda, \mu, z) = (1 - \mu b(z) - \lambda a(z))(1 - \mu b(z)) + \mu a(z)(-\lambda b(z) - \mu c(z)) = 0$$

Oxirgi ifodani soddalashtirsak, (5) kelib chiqadi.

(Yetarliligi) Faraz qilaylik, $z \in \mathbb{C} \setminus [0, 8]$ soni uchun Fredholm determinanti $D(\lambda, \mu, z) = 0$ bo'lsin. Bu holda z soni $h_{\lambda\mu}$ operatorning xos qiymati bo'lishini ko'rsatishimiz kerak. Yuqorida keltirilgan almashtirishlar natijasida $f(p)$ funksiyaga mos ravishda quyidagi (6) bir jinsli chiziqli algebraik tenglamalar sistemi hosil bo'ladi. Uning asosiy determinanti aynan nolga teng ekanligidan sistema trivial bo'lmagan yechimga ega bo'ladi. Demak, $(C_1, C_2) \neq (0, 0)$ mavjud. f yechim va C_1 hamda C_2 koeffitsiyentlar quyidagicha bog'langan.

$$f(p) = \frac{1}{\varepsilon(p) - z} ((\lambda + \mu \cos(x_0, p))C_1 + \mu C_2)$$

Bevosita tekshirish orqali ushbu funksiya

$$(h_{\lambda\mu} f)(p) = z f(p)$$

tenglikni qanoatlantiradi. Demak, nolmas xos funksiya mavjud. Bu esa z sonining xos qiymat ekanligini bildiradi. Oxirgi mulohazalardan z soni $h_{\lambda\mu}$ operatorning xos qiymati ekanligi kelib chiqadi. Teorema isbotlandi.

Fredholm determinantining xossalari. Har qanday $z \in \mathbb{C} \setminus [\varepsilon_{min}, \varepsilon_{max}]$ uchun $\alpha(z)$ noldan farqli ekanligidan

$$D(\lambda, \mu, z) = \alpha(z) \left(\frac{\xi(z)}{\alpha(z)} \mu^2 - \frac{2\beta(z)}{\alpha(z)} \mu + \frac{1}{\alpha(z)} - \lambda \right) = 0$$

o'rniga

$$P_z(\lambda, \mu) := \frac{\xi(z)}{\alpha(z)} \mu^2 - \frac{2\beta(z)}{\alpha(z)} \mu + \frac{1}{\alpha(z)} - \lambda = 0.$$

parabola (μ ning funksiyasi sifatida)ni o'rganamiz.

Biror bir tayinlangan $x \in \mathbb{Z}$ soni uchun

$$R(x, z) = \frac{1}{2\pi} \int_{\mathbb{T}} \frac{e^{i(x,t)}}{\varepsilon(t) - z} dt, \quad z \in (-\infty, 0) \quad (7)$$

funkcijasini ko'rib chiqamiz, chunki $\alpha(z)$, $\beta(z)$ va $\gamma(z)$ funksiyalarni mos ravishda $\alpha(z) = R(0, z)$, $\beta(z) = R(x_0, z)$ va $\gamma(z) = \frac{1}{2}(R(0, z) + R(x_0, z))$ kabi ifodalash mumkin. Quyidagi lemma o'rinlidir.

3-lemma. *Ixtiyoriy tayinlangan $x \in \mathbb{Z}$ uchun $R(x, z)$ musbat va monoton o'suvchi. Shuningdek, $R(x, z)$ uchun quyidagi asimptotik munosabat o'rinli:*

$$R(x, z) = O\left(\frac{1}{|1 - z|^{|x|+1}}\right), \quad z \rightarrow -\infty$$

hamda $\lim_{z \rightarrow 0} R(x, z) = R(x, \varepsilon_{\min})$.

Bu lemmaning analogi isboti bilan birga [5] ishda ko'rishimiz mumkin.

4-lemma. (a) *Ixtiyoriy $z \in (-\infty; \varepsilon_{\min})$ uchun*

$$\mu_2(z) = \frac{1}{\beta(z) + \sqrt{\alpha(z)\gamma(z)}} \quad \text{va} \quad \mu_1(z) = \frac{1}{\beta(z) - \sqrt{\alpha(z)\gamma(z)}} \quad (8)$$

sonlari $P_z(\lambda, \mu) = 0$ parabolaning μ - o'qi bilan kesishish nuqatalari va $A\left(\frac{\beta(z)}{\xi(z)}, \frac{\gamma(z)}{\xi(z)}\right)$ nuqta $P_z(\lambda, \mu) = 0$ parabolaning uchi.

(b) *Ixtiyoriy $\zeta, z \in (-\infty, \varepsilon_{\min})$ ($\zeta < z$) sonlari uchun*

$$\mu_1(\zeta) < \mu_1(z) < 0 < \mu_2(z) < \mu_2(\zeta)$$

va

$$|\mu_1(z)| > \mu_2(z)$$

tengsizliklar o'rinli. Bundan tashqari

$$\mu_1^0 = \lim_{z \rightarrow \varepsilon_{\min}^-} \mu_1(z) = \frac{1}{\beta_0 - \sqrt{\alpha_0 \gamma_0}} < 0, \quad \lim_{z \rightarrow -\infty} \mu_1(z) = -\infty \quad (9)$$

va

$$\mu_2^0 = \lim_{z \rightarrow \varepsilon_{\min}^-} \mu_2(z) = \frac{1}{\beta_0 + \sqrt{\alpha_0 \gamma_0}} > 0, \quad \lim_{z \rightarrow -\infty} \mu_2(z) = +\infty. \quad (10)$$

munosabatlar o'rinlidir.

2-teorema. $\alpha(z), \beta(z)$ va $\gamma(z)$ funksiyalar $z \in (-\infty; \varepsilon_{\min})$ da musbat va monoton o'suvchi. Bundan tashqari

$$\begin{aligned} \lim_{z \rightarrow \varepsilon_{\min}} \alpha(z) &= \alpha(\varepsilon_{\min}) = \alpha_0, & \lim_{z \rightarrow \varepsilon_{\min}} \beta(z) &= \beta(\varepsilon_{\min}) = \beta_0, \\ \lim_{z \rightarrow \varepsilon_{\min}} \gamma(z) &= \gamma(\varepsilon_{\min}) = \gamma_0, & \lim_{z \rightarrow \varepsilon_{\min}} \xi(z) &= \xi(\varepsilon_{\min}) = \xi_0. \end{aligned}$$

limitik va

$$\alpha(z) = O\left(\frac{1}{|z|}\right), \quad z \rightarrow -\infty; \quad \beta(z) = O\left(\frac{1}{|3 - z|^{|x_0|+1}}\right), \quad z \rightarrow -\infty;$$

$$\gamma(z) = O\left(\frac{1}{|z|}\right), \quad z \rightarrow -\infty; \quad \xi(z) = O\left(\frac{1}{|z|^2}\right), \quad z \rightarrow -\infty;$$

$$\frac{\alpha(z)}{\xi(z)} = O(|z|) \quad z \rightarrow -\infty. \quad (11)$$

asimptotik munosabatlar o'rinli.

Isboti: 3-lemma orqali $R(x, z)$ har bir $x \in \mathbb{Z}$ uchun $(-\infty, 0)$ da musbat va monoton o'suvchi ekanligini isbotlangan. U holda $\alpha(z) = R(0, z)$ va $\beta(z) = R(x_0, z)$ tengliklarning o'rinli ekanligidan $\alpha(z)$ va $\beta(z)$ larning mos ravishda musbat va monoton o'suvchi ekanligi kelib chiqadi.

$$\cos^2(x_0, t) = \frac{1}{2}(1 + \cos(2x_0, t))$$

tenglikdan $\gamma(z)$ ni

$$\gamma(z) = \frac{1}{2\pi} \int_{\mathbb{T}} \frac{\cos^2(x_0, t)}{\varepsilon(t) - z} dt = \frac{1}{2} r(0, z) + \frac{1}{2} r(2x_0, z)$$

kabi $R(x, z)$ bilan ifodalash mumkin. Xuddi shunday $\xi(z) := \alpha(z)\gamma(z) - \beta(z)^2$ ekanligidan, uni ham $R(x, z)$ bilan ifodalash mumkin. $R(x, z)$ ning monoton o'suvchi ekanligidan $\gamma(z)$ va $\xi(z)$ larni ham monoton o'suvchi ekanligi kelib chiqadi. Asimptotik munosabatlar ham $\alpha(z), \beta(z), \gamma(z)$ va $\xi(z)$ larning $R(x, z)$ orqali ifodalanishlaridan va 3 - lemmadan quyidagicha kelib chiqadi: \mathbb{T} kompakt bo'lgani uchun $\varepsilon(\cdot)$ chegaralangan, ya'ni $|\varepsilon(t)| \leq C$. $|z|$ ning katta qiymatlari uchun

$$\frac{1}{\varepsilon(t) - z} = \frac{-1}{z} \cdot \frac{1}{1 - \frac{\varepsilon(t)}{z}} = -\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{\varepsilon(t)}{z}\right)^n,$$

qator absolyut yaqinlashadi va $\left|\frac{1}{\varepsilon(t)-z}\right| \leq \frac{C'}{|z|}$. Yuqoridagilardan foydalanib $\frac{1}{\varepsilon(t)-z}$ ning ifodasini $\frac{1}{2\pi}$ ga ko'paytirib \mathbb{T} da integrallasak,

$$\begin{aligned} \alpha(z) &= -\frac{1}{z} \sum_{n=0}^{\infty} \frac{1}{z^n} \frac{1}{2\pi} \int_{\mathbb{T}} (\varepsilon(t))^n dt = \\ &= -\frac{1}{z} - \frac{1}{z^2} \cdot \frac{1}{2\pi} \int_{\mathbb{T}} \varepsilon(t) dt - \frac{1}{z^3} \frac{1}{2\pi} \int_{\mathbb{T}} (\varepsilon(t))^2 dt - \dots \end{aligned}$$

Demak

$$\alpha(z) = -\frac{1}{z} - \frac{3}{z^2} - \frac{21}{z^3} + O\left(\frac{1}{|z|^4}\right), \quad z \rightarrow -\infty$$

yoki

$$\alpha(z) = O\left(\frac{1}{|z|}\right), \quad z \rightarrow -\infty$$

munosabatga ega bo'lamiz. So'ngra $b(z)$ uchun asimptotik yoyilmani topish uchun

$$\beta(z) = r(x_0, z)$$

tenglikdan va 3-lemmadan foydalansak,

$$\beta(z) = O(|1 - z|^{-|x_0|_1 - 1}) = O\left(\frac{1}{|1 - z|^{|x_0|_1 + 1}}\right), \quad z \rightarrow -\infty$$

ekanligi kelib chiqadi. Endi $\gamma(z)$ ning ham $z \rightarrow -\infty$ da asimptotik yoyilmasini topamiz:

$$\left|\frac{\cos^2(x_0, t)}{\varepsilon(t) - z}\right| \leq \frac{1}{\varepsilon(t) - z}$$

munosabatning o'rinli ekanligidan va $\alpha(z)$ ning yoyilmasidan

$$\gamma(z) = \frac{1}{2\pi} \int_{\mathbb{T}} \frac{\cos^2(x_0, t) dt}{\varepsilon(t) - z} = O\left(\frac{1}{|z|}\right), \quad z \rightarrow -\infty$$

kabi ifodalash mumkin. Hamda $\xi(z) = \alpha(z)\gamma(z) - \beta(z)^2$ ekanligidan

$$\xi(z) = O\left(\frac{1}{|z|^2}\right), \quad z \rightarrow -\infty$$

va nihoyat

$$\frac{\alpha(z)}{\xi(z)} = O\left(\frac{|z|^{-1}}{|z|^{-2}}\right) = O(|z|), \quad z \rightarrow -\infty$$

asimptotik munosabatlarning o‘rinli ekanligi kelib chiqadi. Shu bilan teorema to‘liq isbotlandi.

XULOSA

Xulosa o‘rnida shuni aytish mumkinki, yuqorida (3) operatorning spektrlarini o‘rganishda, uning xossalarini bilish juda muhim hisoblanadi. Shundan kelib chiqqan holda, ushbu ishda (3) ning spektrlarni o‘rganish masalasini $D(\lambda, \mu, z)$ Fredholm determinantining nollarini o‘rganish masalasiga keltirildi. (3) operatorning o‘z-o‘ziga qo‘shma ekanligi isbotlandi. (3) operatorning xos qiymatlarga ega bo‘lishining zaruriy shartlari aniqlandi. Fredholm determinantining koeffitsiyentlari uchun z ning turli qiymatlarida asimptotik munosabatlar aniqlandi.

Minnatdorchilik. Ilmiy rahbar sifatida Z.E. Muminovga uning rahbarligi va qo‘llab-quvvatlashi uchun chuqur minnatdorchiligimizni bildiramiz.

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